The Slacker's Guide to Physics: Electricity and Magnetism

Yosun Chang

October 20, 2003

Contents

Chapter 1

An Introduction to The Slacker's Guide to Physics

The idea behind this series is basically encoded in the Slacker's Oath: "I shall always take the path of least action whenever possible." In other words, this book is the text-embodiment of the "least action" - least amount of time spent, energy wasted, etc. - one has to take towards acing this particular academic subject.

I believe that the method of learning promoted by this series - that one should learn certain highly procedure- based methods before attempting to "understand" the material - would allow the following:

- A heightened ability to tackle the traditional problems that fetter many students
- Learning that is directly related to acing the bulk of (most) examinations
- Intimate relationships with some cute (and possibly sexy) equations

Each volume will begin from near-scratch. For example, the E and M volume assumes only that you've taken (or know the bare basics of) single-variable calculus. Although it is highly recommended that students take multivariable calculus before E and M, there are only a few dragons that can't be slayed without. The Math-in-a-Nutshell section should be a sufficient excuse to not let math screw over your physics grade (aside from arithmetic errors and that sort). Also, in case you find me too pendantic in some sections, there will occassionally be "too lazy to read the previous section" chapters that succinctly summarize the main details.[1]

Although the word "slacker" implies that you'd spend your term doing more playing than studying, I cannot promise you an A in your course if you read

[1]Please try to read as much as you can, though. And send me comments (yosun@nusoy.com) whenever I'm unclear or anything, as this is a pre-print. If you want more incentive, I guess I'll put you on my credits page if your comments are good enough. =P

only just this. Reading this will provide you with an alternate and possibly more illuminating method of preparing for examinations and quizzes. However, because this text is focused more on quantitative problems, rather than qualitative methods (although many qualitative characteristics are often derived, or can be taken, from some fundamental quantitative bases), it is a good idea to try to pay attention in lectures and all that (and maybe even occassionally read your main textbook, say on the toilet, perhaps).

–Your Slacker Guru

Chapter 2

Mathematics (of the very helpful kind) in a Nutshell

2.1 Flavors of Derivatives

Once upon a time in algebra, you learned that the slope was defined as rise over run. If you're given the equation of a straight line, say, $y = 3x + 4$, you were supposed to use the formula $y = mx + b$, match coefficients, and find that $m = 3$, therefore, the slope is 3 (because m is defined as the slope). If you were given an equation of a decent parabola, say, $y = x^2 + 3x + 4$, and asked to find the slope, you were supposed to shrivel up into a fetal shape and cry.

Back then, you weren't supposed to know that there were two ways to find the slope. There was just that one formula, and it only worked for straight lines. Only that and nothing else.

So anyway, along comes calculus and the concept of the derivative, a general way to find slopes. By using a simple power rule, you can easily find the slope of parabolas, cubics, 4-power thingies, and so much more. But, apparently, curves have changing slopes. Thus, you needed the idea of the "instantaneous slope." That's actually precisely why derivatives are useful. They generate an equation that defines the slope of the original line/curve at any point.

This section of the book will prep you up on the derivatives required for your E and M course. It'll start with a brief review of partial derivatives, and then it'll plunge into the del operator, which is more or less a shortcut for taking derivatives in three dimensions. DO NOT LET FEAR STOP YOU!!! Chances are, much of the del stuff will become more clear as you proceed onwards with the book.

It's best to get rid of your calculus phobia now, before it's too late. Your physics exams and quizzes will very likely involve some bit of calculus. That's not too bad, basically because you wouldn't want to do it without calculus, anyway. (There are certain things better done in calculus than via algebra.)

2.1.1 Partial Derivatives: The Art of Ignoring

Let's start with derivatives. Nothing nasty like arc-trig. Rather, let's play with those yummy power rules.

Recall from Calculus (or The Slacker's Guide to Calculus: Single-Variable) that:

$$\frac{d}{dx}\, x^2 = 2x \text{ or in general: } \frac{d}{dx}\, x^n = nx^{(n-1)}$$

Basically, "partial derivatives" are extensions of the theory of derivatives from 1-Dimensional calc into multiple dimensions. (We'll focus mostly on 2D or 3D in this text.) The trick is that you differentiate only with respect to the variable indicated by the partial derivative (the curly d - think of it as a faux-multidimensional d) and you ignore the other variables. Thus:

$$\frac{\partial}{\partial x}\, 3x^5y^{69}z^{24} = 15x^4y^{69}z^{24}$$

Notice that the derivative is taken with respect to x, thus only the x term in the equation above is "derivatized" - everything else, all those ugly powers of y and z stay constant. All the rules for derivatives from the 1-D calc course you took last term, or some other time in your dark past, work for partials. (Thus, you can still have those wacky bashes with the ol' product rules, chains, and even the one and only: $(\frac{hi}{ho})' = \frac{hodhi-hidho}{hoho}$.)

Q. But, O Great Slacker Guru, what does this mean? I mean, it's fun taking mindless power-rule derivatives and all, but... A. When you take something with respect to x, while ignoring the y and z variables (components, actually), for example, that means you're only finding the slope of the function in its x component. The derivative with respect for x would (in general) not apply to the slope for the y and z components.

2.1.2 The Del "∇" Operator

The Del operator is a group of partial derivatives. Although it might look scary, it's really just a shorthand for stating derivatives with respect to different coordinate systems. Different coordinate systems are really very cool[1]; they allow you to specify your position in an objective unambiguous way. Your choice of coordinate system might make certain problems way easier, as we shall see quite soon.

Let's take a look at the Cartesian coordinate system first. It's the basic x-y-z thing you've known for a while.

Suppose you want to take derivatives of a function $f(x, y, z)$ with respect to x, y, and z. You can do this two ways:

[1]see "Moving the Origin"

- The long way: $(\frac{\partial}{\partial x}\,\hat{\mathbf{x}} + \frac{\partial}{\partial y}\,\hat{\mathbf{y}} + \frac{\partial}{\partial z}\,\hat{\mathbf{z}}\,) \star f(x, y, z)$

- The "del[2]" (∇) way: Define $\nabla = \frac{\partial}{\partial x}\,\hat{\mathbf{x}} + \frac{\partial}{\partial y}\,\hat{\mathbf{y}} + \frac{\partial}{\partial z}\,\hat{\mathbf{z}}$ Thus, you can state everything in the "long way" as: $\nabla \star f(x, y, z)$

Notice that I've placed a $\star$ there, which I'll get back to. And then, notice that I've introduced three unit vectors[3] $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$. (They usually refer to these unit vectors as $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$, respectively, but it's the same meaning, just different "names.") These unit vectors are mutually perpendicular to each other (they form the axes). They are therefore linearly independent bases[4] that span all of 3-space[5]; from a linear combination[6] of these, you can construct any other vector (in 3-space).

Read the footnotes to the last paragraph. Then, memorize the last two sentences. They're good for impressing certain people at cocktail parties. Seriously, you should try this.

Anyway, I put a $\star$ there because of those unit vectors. Vectors are values that contain both direction and length. In the case of unit vectors, the lengths are all unitary, i.e., 1. It is only the direction that is of quintessential significance in these unit vectors. Vectors have direction (aside from the trivial sense of ordinary numbers that are just negative or positive) because they have multiple components.

The cartesian vector has three components: the x, y, and z components. Different mixes of values yield different vectors. Multiplying two vectors in different ways also yield different results. Sometimes vector multiplication commute, and sometimes, they do not.

There are three ways to multiply vectors:

- The first way involves only one vector. You multiply a scalar α and a vector $\vec{A}$ and you get $\alpha\vec{A}$

- The second way is called the "dot $\cdot$ product." (AKA the "scalar product" - the end result of multiplication of these two vectors yields a scalar) You multiply the vectors $\vec{A}(a, b, c)$ and $\vec{B}(d, e, f)$, in this particular way, to get their dot product: $\vec{A} \cdot \vec{B} = (ad + be + cf) = \parallel \vec{A} \parallel \parallel \vec{B} \parallel \cos\theta$. You get a

[2]Del is not a vector. It acts on vectors and numbers and "transforms" them according to its definition as stated above.

[3]Unit vectors will be denoted in bold face with hats. Regular vectors will just have arrows on top of them.

[4]They are independent in that you need only a linear combination of - say - the $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$, unit vectors to uniquely describe any vector in the xy-plane.

[5]3-space is short for 3D-space

[6]A linear combination is just a bunch of vectors each multiplied by some scalar "ordinary" number.

scalar from the dot product of two vectors (after multiplying the A_x[7] with B_x and A_y with B_y, etc.).

- The third way is called the "cross $\times$ product." (AKA the "vector product" - the end result of multiplication of these two vectors yields a vector.) You multiply the vectors $\vec{A}(a,b,c)$ and $\vec{B}(d,e,f)$ to get $\vec{A}\times\vec{B} = det(\hat{\mathbf{u}}, \vec{A}, \vec{B})$ where $\hat{\mathbf{u}}$ represents the unit vectors. Thus the cross product of A and B is mutually perpendicular to both and can be calculated in Cartesian coordinates as this:

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ a & b & c \\ d & e & f \end{vmatrix} = \hat{\mathbf{x}}(bf-ce) - \hat{\mathbf{y}}(af-cd) + \hat{\mathbf{z}}(ae-bd)$$

  where the magnitude of this vector is equal to $\parallel \vec{\mathbf{A}} \parallel \parallel \vec{\mathbf{B}} \parallel \sin\theta$

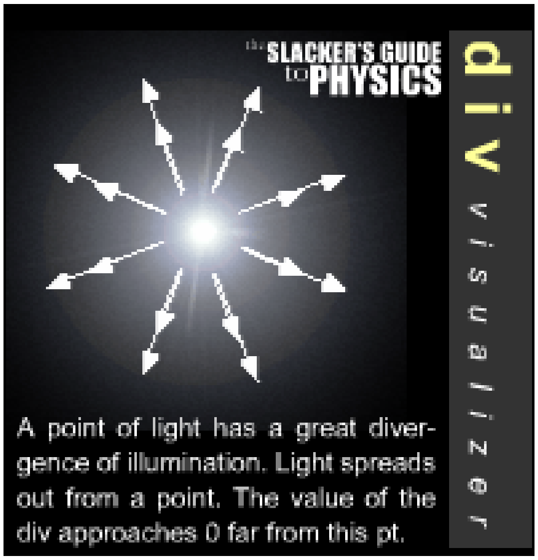
  I put the unit vectors in front of the values because a scalar multipled by a vector commutes, and also, you might find it easier in your calculations in the future to write vector products as such. The $\parallel$ "cages" around $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ make scalars out of the two vectors like this: (eg) $\parallel \vec{\mathbf{A}} \parallel = \sqrt{a^2+b^2+c^2}$. Theta θ is the angle between the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

Back to our old ∇. The ∇ isn't really a vector, but it has components. Del doesn't mean anything by itself, but, like vectors, it does not necessarily commute, hence why i had a $\star$ earlier. So... you can multiply it with another vector in similar ways:

- grad $f = \nabla f$ ... Del isn't really a scalar, but f is a scalar, in this case. Grad makes a vector out of f. Thus: Suppose $f = x^2 + xy^3z^4$. The grad operation would transform that scalar function into a vector function. A once direction-less "lost" function would be direction-ized! It would do so like this: $\nabla f = \nabla(x^2 + xy^3z^4) = \hat{\mathbf{x}}(2x + y^3z^4) + \hat{\mathbf{y}}(3y^2xz^4) + \hat{\mathbf{z}}(4z^3xy^3)$. **Conceptual Meaning:** The gradient ∇f points in the direction of maximum increase of the function f.

- div $f = \nabla \cdot \vec{\mathbf{f}}$ ... The dot product of del and a vector (del "dot" $\vec{\mathbf{f}}$) yields a scalar. Suppose $\vec{\mathbf{A}} = < x^2y, xyz, x^3yz^4 >$. Then $\nabla\cdot\vec{A} = (2xy + xz + 4z^3x^3y)$. **Conceptual Meaning:** The divergence is a measure of how much a vector "spreads out" from a particular point.

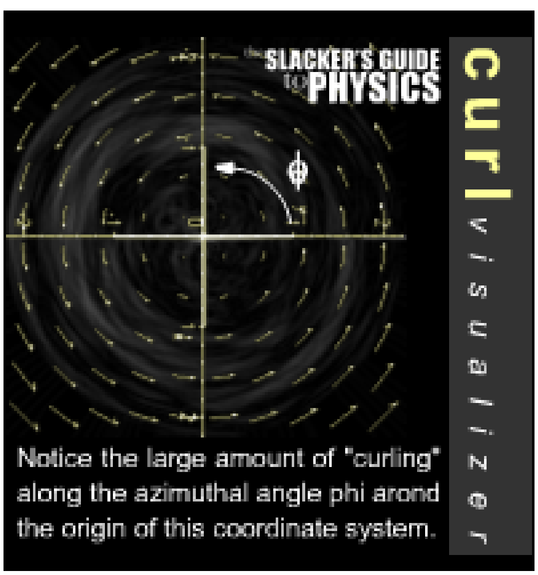
[7] where A_x represents the x component of A, rather than the derivative of A with respect to x. I will always specifically write $\frac{d}{dx}$, etc.



- curl $f = \nabla \times \vec{\mathbf{f}}$ ... The cross product of del and a vector (del "cross" $\vec{\mathbf{f}}$) yields a vector. Suppose $\vec{\mathbf{A}} =< a, b, c >$, where a, b, and c are functions of x, y, and z.

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a & b & c \end{vmatrix} = \hat{\mathbf{x}}(\frac{\partial c}{\partial y} - \frac{\partial b}{\partial z}) - \hat{\mathbf{y}}(\frac{\partial c}{\partial x} - \frac{\partial a}{\partial c}) + \hat{\mathbf{z}}(\frac{\partial b}{\partial x} - \frac{\partial a}{\partial y})$$

Conceptual Meaning: The curl is a measure of how much a vector "curls around" a particular point.



Incidentally, the divergence of a curl is zero because if you look at the diagrams above, you can see that something with only a curl around a point does not have a divergence at that point.

2.2 Too Lazy to Read the Previous Section

OK. You don't really have to understand del just yet. Working through the rest of the material of the text should give you the same thing. For the time being, just memorize the following relations and then flip onto the next modicum:

- $\nabla = \frac{\partial}{\partial x}\,\hat{\mathbf{x}} + \frac{\partial}{\partial y}\,\hat{\mathbf{y}} + \frac{\partial}{\partial z}\,\hat{\mathbf{z}}$
- grad $f = \nabla f = \hat{\mathbf{x}}\frac{\partial f}{\partial x} + \hat{\mathbf{y}}\frac{\partial f}{\partial y} + \hat{\mathbf{z}}\frac{\partial f}{\partial z}$
- div $\vec{f} = \nabla \cdot \vec{\mathbf{f}} = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}$ See Footnote[8]
- curl $\vec{f}$ is as below

$$\nabla \times \vec{f} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_x & f_y & fz \end{vmatrix} = \hat{\mathbf{x}}(\frac{\partial c}{\partial y} - \frac{\partial b}{\partial z}) - \hat{\mathbf{y}}(\frac{\partial c}{\partial x} - \frac{\partial a}{\partial c}) + \hat{\mathbf{z}}(\frac{\partial b}{\partial x} - \frac{\partial a}{\partial y})$$

2.3 Misc. Tricks

2.3.1 Moving the Origin

You can move your origin anywhere, as long as you are consistent. For example, suppose I want to find the distance from A to B:

$$A(5,4)\ ;\ B(5,10)$$

I can move the origin to either A or B. Suppose I move it to A. Then:

$$A(0,0)\ ;\ B(0,6)$$

Note that these are two equivalent systems as long as A and B are spaced $(0,6)$ apart. It's all relative.

This is an overtly simplified example of why you would want to move your origin, and I don't blame you for thinking me a total bore for tellng you this. Anyway, you can easily create your own coordinate system this way. It's all quite cool.

Oh, also, you can create coordinate systems by redefining your axes. (And, I guess if you're of the "hardcore" math type, you would complain that moving the origin isn't really creating a new coordinate system. But anyway, I'm referring to the pedestrian usage of coord sys.) Suppose you want to create a coordinate system on the surface of a sphere, for example, then you would want to define your axes accordingly to simplify the problems on the sphere. Se the next section for details...

[8]note that f_x denotes the x component of x, etc.

2.3.2 Rotating the Coordinate System

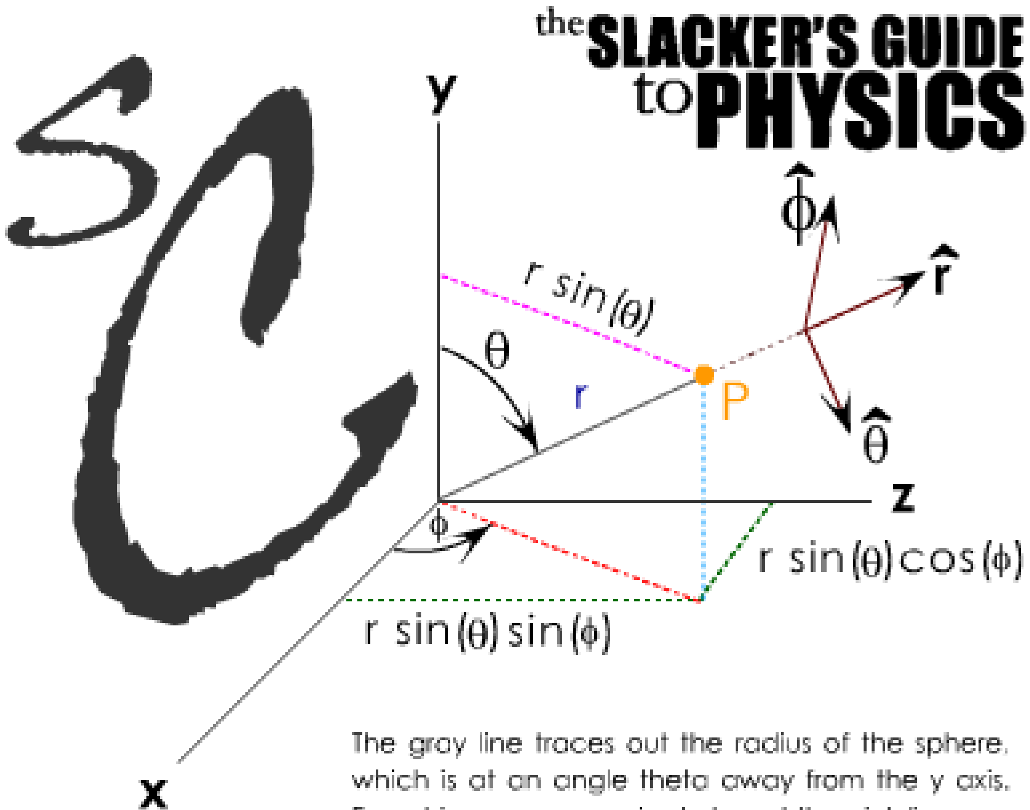
Ya. So you know you can move the origin. Well, you can rotate your axes too. See the "Inclined Planes" subsection in Components earlier in this chapter.

2.3.3 Spherical Coordinates are Soooo Sexy!

Spherical coordinates (SC) may look formidable, if not frightening, but that's only if you let the explicit usage of various angles get to you. SC is actually quite sexy, once you get to know it.

You can form two kinds of relationships with SC:

- **The Shallow-Hal-Wannabe Kind**- You memorize the following transformations from your homebase Cartesian coordinate system to spherical coord sys. This is **what** SC is; unless you're a math major or taking an upper div course in science, it's really only this and nothing more:
  - $x = r \sin\theta \cos\phi$
  - $y = r \sin\theta \sin\phi$
  - $z = r \cos\theta$
  - where $\theta \in (0, \pi)$ and $\phi \in (0, 2\pi)$
- **The Meaningful Kind**- You get to know **who** SC is. You have to form a Shallow Relationship first, though, viz., what is SC? Then, you should ask yourself: **How** did SC come to be? (SC's roots and origins are in Meaningful Projections.) You consider **why** anyone would bother finding out how SC came to be and **when** you'll need the sexy entity known as SC. And finally, once you fully know SC, you can just forget everything above, except the when part, and just start stalking SC That is, you should know **where** SC is at any time (whether on the formulae sheet on your exam or on the inside cover of your textbook, etc.) But, I warn you, the where-part might become redundant, eventually, as through constant usage, you may know SC so well, you'll wind up memorizing SC. That's a pivotal stage you don't reach in many other relationships; that just goes to show how much cooler SC is than everything and *everyone else.*



The gray line traces out the radius of the sphere, which is at an angle theta away from the y axis. From trig, one can project r to get the pink line. Because the Pink line is parallel to the red line, the red line is also equal to r sin theta. The red line is the hypothenuse of the triangle formed by either green line, on the xy plane. It's projection again, the hypothenuse is equal to r sin theta in this case, and the x and y component of that follow in the usual way. Finally, note the spherical unit vectors: r, z, phi, in crimson. They are mutually perpendicular to each other and together form a basis.

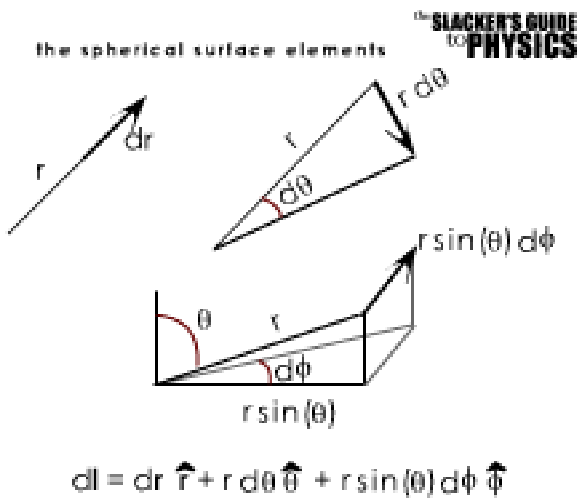
The above is what SC really looks like. It shows what the x, y, and z values coorrespond to. There's also the restriction for $\theta \in (0, \pi)$ and $\phi \in (0, 2\pi)$. Slackers are oft too lazy and think that the angles go from 0 to 2π in both. Well, that won't work, because then you would be counting the "height" twice. Think of it like this: suppose you wanted to measure your own height, going from 0 to π is like going from head to toe, if you assign your head the value $\theta = 0$ and your toes the value $\theta = \pi$. If you went all the way to 2π, you would be counting your height from head to toe plus your height from toe to head. There'll be trouble in your caboose!

That's **how** SC came to be. SC can be very useful especially **when** you're working with spheres. Say, for example, you wanted to find the surface area of a sphere the hardcore way via multiple integrals. You can tough it out via Cartesian coordinates, or you can do it the easy way via a coordinate system designed especially for such problems. (You can find this example in your Calculus book.)

If you're confused at how I got the x,y,z correspondence, maybe you've forgotten how to resolve a vector into components. (To wit: How did you solve those incline plane probs from Mechanics?) Anyway, resolving a vector into components is really just projecting it onto the axes of your choice. Usually, that's the x and y axes. If you know theta and the hypothenuse, life is good. The x component would be just $h \cos\theta$ while the y component would be $h \sin\theta$ ... this is providing that your angle is adjacent to the x axis. It's the other way around if it's adjacent to the y axis.

The spherical unit vectors are pretty important. For LD, you don't have to necessarily coorespond them to their x, y, and z parts, but you should conceptually understand one of them: The r component protrudes in the radial direction; it will always be pointing away from the origin in a sphere.

Incidentally, the spherical surface element $d\mathbf{a}$ (where a is a vector normal to the surface) can be constructed from a multiplicative mix of any two of the



below.

2.3.4 Cylindrical Coordinates!!!

Chapter 3

The Electrostatics Introduction

The reason why they have electromagnetic theory is basically to solve problems involving charges; the reason why you have to take this course if you are a science major or engineer is because it introduces you to one of the fundamental theories that govern all of the modern technological world. The material you learn in this course will probably seem pointless to you, as there are very few actually practical "real-life" applications, but in order to truly understand how anything involving electricity works, you need to know this stuff.

Although the Coulomb Force law looks a lot like the Gravitational Law from Mechanics, the charges behave differently than people, or many ol'masses, for the most part. There are certain explicitly defined rules in electromagnetism that make charges and things a lot easier to predict, relative to people. Furthermore, we know that the electrostatic field is always conservative. That's not true for many instances in real life; for example, there will always be air friction, etc. Thus, you can simplify many-a-nasty integrals by knowing that any path from A to B will be equal. Electricity and Magnetism is thus an easier course than Mechanics, in my opinion. But, it's more from this, than from what I said before: if you know your mathematics, then the answers you want are practically given to you via Maxwell's Equations (below). If not, then this book will hopefully beef up your (relevant) math skills so that, you, too, can ace EM without much work.

For example, an ideal test charge Q isolated within its own universe will produce an electric field of $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}}$. This will always be the case. Even if it is Tuesday.

The laws of electromagnetism strictly define the behavior of charges. Thus, unlike certain other branches of physics, electromagnetism is completely deterministic. In the ideal world of textbook (and exam) problems, the 2 (of 4 total) Maxwell equations summarize the life of every single electrostatic charge in the universe:

$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ and $\nabla \times \vec{E} = 0$

The above are the relevant Maxwell's equations for electrostatics - the state of non-moving charges. It's also the state most first year undergraduate Electricity and Magnetism course starts at.

The first equation states that the source of the field is in the charge. Rho is the charge distribution.

The second equation states that the electrostatic field is conservative. There is no such thing as "electrostatic friction." Thus, when you calculate the work done to get from point A to point B, you get the same answer no matter that path you choose (if you set up the line integral right!). Also, because the Electrostatic field is conservative, $E = -\nabla V$, i.e., the Electric field can be defined as the negative gradient of a potential. This will come in very handy when you're asked to find electric potentials, especially when you're given the electric field.

If you know the Divergence Theorem (in the Math section), you can transform the first equation into integral form like this:

Divergence Thrm: $\int\int\int(\nabla \cdot \vec{E})dV = \int\int \vec{E} \cdot \vec{da}$ You know that $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} = \frac{\int\int \vec{E} \cdot \vec{da}}{dV}$ You get the last part by solving for $\nabla \cdot \vec{E}$ in the Divergence Thrm.

You can now leave out the $\nabla \cdot \vec{E}$, as you're seeking an integral form, and that's the differential form. And, you can multiply the dV on the denominator to simplify things on the right hand side. Thus, you get: $\frac{\rho dV}{\epsilon_0} = \int\int \vec{E} \cdot \vec{da}$

Now, I'll review the precise meaning of ρ. It is defined as the $\frac{charge}{unitvolume} = \frac{q}{V}$. Then, $q = \rho dV$ Thus, you can simplify the right hand side into: $\frac{q}{\epsilon_0}$ The end result is this:

$$\int\int \vec{E} \cdot \vec{da} = \frac{q}{\epsilon_0} \tag{3.1}$$

You now have Gauss' Law in integral form. This is where all the fun shall begin...

Chapter 4

Electrostatics

4.1 Gauss' Law

$$\oint \vec{E} \cdot d\vec{a} = \frac{q_{in}}{\epsilon_0} \tag{4.1}$$

Equation 4.1 is Gauss' Law in integral form [1]. The left side of the equation is a scalar/dot product of two vectors; it corresponds to the scalar on the right side. In words, it reads: The closed integral of the electric field $\vec{E}$ dotted with an area element $d\vec{a}$ (Electric Flux) is equal to the charge enclosed within (i.e.,"inside") the area element divided by the permittivity of free space: ϵ_0

By closed integral, I mean the surface should "trap" the charge in question within itself; it should not have a hole. Because the left side is a dot product, $\vec{E}$ would be nonzero only where $\vec{da}$ is nonzero. (This relationship will always hold in context of this Gauss' Law equation, but it might not always work in other cases!) The right side of the equation is the net charge enclosed within the area element on the left side. In a sense, it's kind of like a body-count. You can see the left side as a warehouse-surface enclosing a quantum [2] amount of hostages. Oh, the warehouse walls emanate a totally tell-tale Electric field. You're actually some FBI agent trying to negotiate with these criminals who dunno Electrostatics. They claim that they have twenty q's in there, but from the left side of the equation, the surface of the warehouse, you easily see that they're bluffing. Their going-rate is $1,000,000/person - er charge. You scream at the top of your lungs on your loudspeakers that they're obviously bluffing (and electro-illiterate). They only have 5 people - er, charges - in there, thus you will only pay a ransom of

[1]Maxwell's Equations was stated previously terms of del's, which is the differential form. This integral form can be derived by using the Divergence Theorem.

[2]quantum refering to an integer amount... we're not going to be too gruesome as to assume the warehouse contains fractional amounts of people. Also, the quantum requirements is interesting because real charge is always quantized... the continuous distribution questions are thus really only approximations. They aren't as "precise" and "godly" as the mathematics might imply.

\$5,000,000. You just saved \$15,000,000. Wow! Of course, real warehouses are not as convenient as Gaussian surfaces ($\vec{da}$), at least not yet. And then, real FBI agents would probably blast their way in with machine guns or napalms or bazookas or teleportation devices and end up not paying anything...

If you're pissed that I'm teaching you Gauss' Law when your course is only on "continuous charge distributions," all I can say is bare with me... you just might find Gauss' slacker methods helpful in, at the very least, checking some of your messy continuous charge things...

Gauss was probably one of the coolest slackers. This particular law of his makes finding the Electric field of a sphere (anything with certain symmetery you shall see quite soon), even those plastered with non-uniform charge distribution, grotesquely easy. But, before I show you that, I'll begin with its most fundamental application:

4.1.1 Illumination from a Point Charge

- Example: Finding the Electric Field $\vec{E}$ a distance r away from a point charge q.

  Problem: Suppose you have a point charge q located out in the middle of nowhere (like, really, it's a vacuum, total free space, so you can use ϵ_0). For whatever reason, you need to find the $\vec{E}$ field at a distance r away from a point charge.

  Answer: You can do this two ways. You can get this directly from the most basic form of Coulomb's law. (to wit: $\vec{E} = \frac{q\hat{\mathbf{r}}}{4\pi\epsilon_0 r^2}$) or... You can get this from Gauss' Law, like so: $\vec{E} \cdot (4\pi r^2 \hat{\mathbf{r}}) = \frac{q}{\epsilon_0}$

  The area element in this case would be the surface area of the sphere. This is so because of the nature of the field lines. Like in the divergence diagrams given in the Math-Nutshell chapter, the Electric field of this point charge has great divergence. It's been pre-determined that Electric field lines begin in positive charges and end in negative charges.

  Because there is no negative charge in sight from here until infinity (or the end of our lil vacuum space), it looks like the field lines from q will have no curl, but a huge divergence (it's like light streaming out from the sun or some dominant luminous entity) due to the fact that its original source (to wit: q) is clearly identified.

  The divergence will be only in the radial direction, as the change in field intensity will be constant in all the other directions. The Electric field is therefore zero in the other directions.

  The radial direction is the one that is always perpendicular to the area of the sphere that encloses the charge. Define the direction of the area element as the radial direction (think of it like this: for any infinitesimally tiny patch on the sphere, the vector that is perpendicular to the surface will always be pointing radially away).

Thus, the equation above reduces to this scalar: $E(4\pi r^2) = \frac{q}{\epsilon_0}$.

From this scalar equation, we can solve for E, and we get $E = \frac{q}{4\pi\epsilon_0 r^2}$

We can direction-ize E, transforming it into $\vec{E}$, by retracing our steps. We defined $\vec{da}$ to be nonzero in only the radial direction/component, therefore E is consequently zero in all other components (multiply the dot product out!). Thus $\vec{E} = \frac{q\hat{\mathbf{r}}}{4\pi\epsilon_0 r^2}$

So, from the example above, we have a basic heuristic for solving Gauss' Law problems. And... It's hella titeXD

To recap: $\vec{E}$ dot $\vec{da}$ contains only the components of $\vec{E}$ and $\vec{da}$ that are in the same direction. The direction of the area element is thus taken as always perpendicular to the actual surface. The surface element must enclose all of the charge indicated on the right side of the equation. Once you've plugged in the right values, you can solve for E. In general, it is good to always set up your equation in the same format as Equation 4.1.

4.1.2 Multiple Charges and Electric Flux!

Further clarification: Suppose we have four charges, q1, q2, q3, and q4. This time, we're seeking the Electric flux in the region, that is: the fluxuation of the electric field lines ($\Phi = \oint \vec{E} \cdot \vec{da}$). So, formally, the problem goes like: Find the electric flux in a sphere that spans all space of these four charges. Assume they're all in vacuum.

The first thing to do is to state Gauss' Law in its most general form:
$\oint \vec{E} \cdot \vec{da} = \frac{q_{in}}{\epsilon_0}$ (in general)

Then, you think a bit about what each of the variables above mean:

In this case, the charge enclosed would be all four of the charges. This is so because the Gaussian surface spans all of space (at least relative to this particular region). Therefore, there is no danger of leaving any one charge out! Because we only need to find the electric flux, we can worry about just one side of the equation and forget the rest. In this case, we're too lazy to figure out the Electric field, thus we'll just do the right hand side.

$\Phi = \oint \vec{E}_1 \cdot (\pi r^2 \hat{\mathbf{r}}) = \frac{q_1+q_2+q_3+q_4}{\epsilon_0}$

Thus $\Phi = \frac{q_1+q_2+q_3+q_4}{\epsilon_0}$ and that's the answer!

4.1.3 Sphere 0: Uniformly Distributed Surface Charge

We shall now begin the Trilogy of the Spheres. This is the Prologue. Like many good fantasy trilogies, you have to read this to get the rest. So... Read this sub-section!

In another tone, the purpose of Gauss' Law is to escape the nasty integrals of the continuous-charge distribution questions. This example should prove it (at least for one case):

Suppose charge q is uniformly distributed on the surface of a sphere of radius R. (You can define $\sigma = \frac{q}{4\pi R^2}$ as the surface charge density.) Find the Electric field everywhere.

The left hand side of Gauss' Law is trivial. It's just $E(4\pi r^2)$ (look at the interactive diagram for divergence available at http://slacker.yosunism.com - remember, the Electric field has extreme divergence from the center of the sphere, thus the field is going in the same direction as the radius, always... so, we have only the E_r component multiplied by the dA_r comp.) The other components cancel because the field is only in the radial direction, thus $\hat{\theta}$ and $\hat{\phi}$ don't matter. But then, if you define the vector area to be in the direction that is always normal to the surface, then there wouldn't be the other two components in the first place.

The right hand side isn't so bad, either. It's just that you have to realize that the field inside and the field outside will be different.

Inside the sphere ($r < R$), there is NO charge. Thus $q_{in} = 0$. And, according to Gauss' Law, the field is thus 0 inside.

Outside, the problem reduces to that of a point charge. You have just $\frac{q}{/}\epsilon_0$ on the right hand sand. ... I think you can take it from here. (If not, read the previous section on Gauss' Law.)

4.1.4 Sphere 1: Non-uniformly Distributed Surface Charge

Suppose there is a wacky non-uniformly plastered charge distribution $\sigma = 3b^2$, where b is a function of θ and ϕ. This is stuck onto the surface of a sphere of radius R. Find the electric field everywhere. You may express charge in terms of an integral.

The left hand side of Gauss' Law is trivial. It's just $E(4\pi r^2)$ (remember, the Electric field has extreme divergence from the center of the sphere, thus the field is going in the same direction as the radius, always... so, we have only the E_r component multiplied by the dA_r comp.) The other components cancel because the field is only in the radial direction, thus $\hat{\theta}$ and $\hat{\phi}$ don't matter, as. But then, if you define the vector area to be in the direction that is always normal to the surface, then there wouldn't be the other two components in the first place.

Inside ($r \leq R$), it's 0, as there is charge only on the surface. No charge enclosed always means the electric field is zero whenever Gauss' Law is involved.

Outside: To solve for the right hand side, you'll need to "think" in terms of q.

$$q = \int_0^{2\pi}\int_0^{\pi} 3b^2 R^2 \sin\theta d\theta d\phi \tag{4.2}$$

This relationship works because the surface charge density σ is defined as $\frac{charge}{surface area}$. Thus, you can solve for charge (in order to plug it into Gauss' Law). (Note that I've used the standard spherical surface element R^2 (see the Math section if you want to know how I got these), where R is constant in this case, as it is on the surface of the sphere, where $r = R$ exactly.)

You plug this into Gauss' Law: $E(4\pi r^2) = \frac{\int_0^{\pi}\int_0^{2\pi} 3b^2 r^2 \sin\theta d\theta d\phi}{\epsilon_0}$

You solve for E. It's still in the radial direction. Don't worry about the fact that theta and phi are involved. The dot product will come out 0 for the components other than the radial one, anyway. But then, if you had that worry, you still don't understand the concept of vector area - it's only the Gaussian surface you have to worry about, and that stays the same for all these sphere questions. The wacky charge distribution can really go to h - e - double hockey sticks, for all that matters here.

4.1.5 Sphere 2: Uniformly Distributed Volume Charge

Sphere of radius R. Uniformly distributed volume charge density. Total charge is q. Find $\vec{E}$ everywhere.

The left side of Gauss' Law is trivial. It's just $E(4\pi r^2)$ (remember, the Electric field has extreme divergence from the center of the sphere, thus the field is going in the same direction as the radius, always... so, we have only the E_r component multiplied by the dA_r comp.) The other components cancel because the field is only in the radial direction, thus $\hat{\theta}$ and $\hat{\phi}$ don't matter, as. But then, if you define the vector area to be in the direction that is always normal to the surface, then there wouldn't be the other two components in the first place.

For outside the sphere, the right side is trivial. The field reduces to that of a point charge.

For inside the sphere, it's a bit different...

The right side can be done with some stoiciometry:

We want to find q_{in} given a uniform distribution.

Uniform distribution means the density is always constant: Thus

$$\frac{q_{in}}{\frac{4}{3}\pi r^3} = \frac{q}{\frac{4}{3}\pi R^3} \tag{4.3}$$

The RIGHT side (in this case) is the density of total charge over total volume. The left side is the density of the charge inclosed by the particular volume. Here, $r \leq R$. You'll see that at $r = R$, the answer is the same as that for $r > R$.

You solve for q_{in} in the equation there, and you plug it into Gauss' Law: $E(4\pi r^2) = \frac{q}{\epsilon_0}\frac{r^3}{R^3}$ for $r \leq R$, then $E = \frac{q}{4\pi\epsilon_0}\frac{r}{R^3}$. You want a direction for E? Well, that should be trivial by now. It's in the radial direction, so stick on a $\hat{\mathbf{r}}$ if it makes u feel better.

4.1.6 Sphere 3: Non-uniformly Distributed Volume Charge

Sphere of radius R with non-uniform volume charge density $\rho = \beta r^7$. where β is some constant. Find the electric field everywhere in terms of β, r, and the fundamental constants.

You should be pretty used to this by now. Anyway, here goes:

Inside: $q = \int\limits_0^r \beta r^7 r^2 \sin\theta dr d\theta d\phi$ note that it's 0 to r, where r can be any radius number less than R. This is to indicate the volume within, of course.

Outside: $q = \int\limits_0^R \beta r^7 r^2 \sin\theta dr d\theta d\phi$

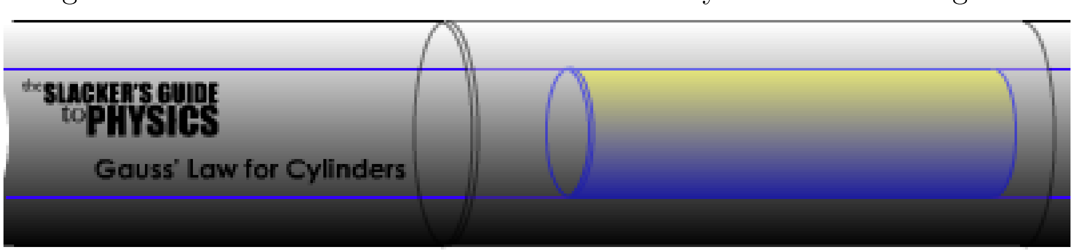
And, the rest is just plugging it into Gauss' law. I have used spherical coordinates in this case, hence why there are $r^2 \sin\theta$ along with the differential's.

4.1.7 Cylinders!

If you're getting sick of spheres or getting way too excited over passing spherical Gaussian surfaces, I have a treat for you!

Recall Gauss' Law: $\oint \vec{E} \cdot d\vec{a} = \frac{q_{in}}{\epsilon_0}$

Then, visualize an infinitely long cylinder in your mind's eye. Imagine that the charge which produces the electric field has been "stretched" so as to be perfectly parallel to the cylinder's longitudinal axis. Moreover, imagine that the charge is concentrated at the exact center of the cylinder. Something like this:



suppose you were superman and could easily see through an infinitesimally small segment of the infinitely long solid cylinder. there would be some sort of "blue" charge enclosed by the gray Gaussian Surface. the field would be radial.

The field would be radial all the time. It would have no other components. Therefore, the surface element $d\vec{a}$ must have a radial component... As we know the field is obviously not zero.

Recall the forumla for area: length × width Recall the formula for the surface area of a cylinder: $2\pi rl$ where $2\pi r$ is the width (or actually, the circumference of the circle - this is actually what you get when you make a cylinder out of any flat piece of paper) and l is the length.

Thus: $d\vec{a} = 2\pi r\hat{\mathbf{r}}l$

Taking the dot product of the left side, you get $E(2\pi rl)$

The right side will vary depending on whether you have uniformly-distributed surface charges, non-uniformly distributed surface charges, uniformly-distributed volume charges, and non-uniformly distributed volume charges. The method for chugging out the results for all those is similar to that for the Sphere Trilogy. I'll outline the few differences, you fill in the blanks from refering to the epic story of the Sphere Trilogy a few pages ago.

For all these, assume the sphere is of radius R.

If the cylinder has non-uniformly distributed surface charge $\sigma = b^3$, where b might be some wacky function of θ and l ("height"): This is how you find the charge inside. For outside, change the limit r to R.

$$q_{in} = \int\int b^3 dA = \int_0^{2\pi}\int_0^r b^3 r dr d\theta \tag{4.4}$$

where I have used the cylindrical area element.

If the cylinder has q uniformly distributed over its volume: $\rho = \frac{charge}{volume}$ Recall that volume = base × height

Thus:

$$\rho = \frac{q_{in}}{\pi r^2 l} = \frac{q}{\pi R^2 l} \tag{4.5}$$

Solve for q_{in} and plug.

If the cylinder has non-uniformly distributed volume charge $\rho = \beta^3$, where β might be a crazy function of anything. Find the Field produced by a segment of length/height h. This is how you find the charge inside. For outside, change the limit r to R.

$$q_{in} = \int\int\int \beta^3 dV = \int_0^h\int_0^{2\pi}\int_0^r \beta^3 r dr d\theta dl \tag{4.6}$$

where I have used the cylindrical area element. Do the usual, and plug in q_{in}

4.1.8 A Plain Ol'Infinite Plane

It goes on and on forever... Also, it's infinitely thin.

The Gaussian surface can therefore be just two pieces of "flat paper." Their specific area is usually belittled by the mere mention of A. We'll do that here, too.

Recall Gauss' Law: $\oint \vec{E} \cdot d\vec{a} = \frac{q_{in}}{\epsilon_0}$

The dot product of the left side goes like: $|E(above)A|+|E(below)A| = 2|E|A$ (remember that A is the surface element perpendicular to the surface area, so it's just the normal to the plane. And, we're taking the absolute value here, so it's additive rather than zero, as E will inevitably point in opposite directions above and below.)

Now, this problem has some intimate ties with direction. For example, the field will likely have a different value above and below the plane. Why? Well, the normal vector (i.e., the direction of the area element) will vary above and below. This also makes sense because, again, the field would originate from the charge. (The vector would start pointing away out into vertical infinity from the surface of the horizontal plane.) And the charge will have to be plastered either uniformly or non-uniformly over the plane surface. (The latter case is extremely unlikely to be a test prob, for now.) Thus, the only original source of the electric field would be from the plane, hence its field direction vector's "origin" there.

The right hand side is just $\frac{q_{in}}{\epsilon_0}$, where q_{in} varies depending on yoru charge distribution. The methods are similar. If you ever need to integrate, you can just use Cartesian, which is natural for planes.

4.1.9 Conductors and Insulators

In electrostatics, charge in a conductor would be found only on the surface. Thus, the only volume charge densities that are possible must be insulators. Insulators don't conduct; they're the opposite of conductors. Therefore, they can't carry the charge around; thus, once you put a charge there, it pretty much stays there. That's why there are non-uniform volume densities in electrostatics, in the first place. That's basically it. You might have a quiz/test question that attempts to trick you (and scare you) by saying, for example, "Find the field inside a spherical conductor of radius R, plastered on the surface with total charge q." Of course, that's just finding the field inside the surface-charge density problem; there is no charge inside: the field inside is zero. The field outside reduces to that of a point charge.

Anyway, in this case, you should be able to translate easily between the qualitative and quantitative descriptions. Example: Qualitative Description of spherical insulator with nonuniform charge means the quantitative non-uniform volume charge density.

4.2 Too Lazy to Read All That About Gauss' Law

Gauss' Law in integral form is:

$$\oint \vec{E} \cdot d\vec{a} = \frac{q_{in}}{\epsilon_0} \tag{4.7}$$

That's all there is to it. But, if you don't know vector calculus very well, applying it might be another thing.

Here's the lowdown on application:

- Determine the origin/source of the field. That's usually in a charge distribution of some sort. Remember the fact that the electric field diverges positively from any positive charge. Because you know about dot products from the math section of this lil ol'book, you can easily ignore the other components of the area element ($\vec{da}$) that are not in the same direction as the field. (To wit: otherwise, the Gauss' Law equation would not hold)
- Take the area element that's normal to the Gaussian surface (also represented by $\vec{da}$). The Gaussian surface should be perfectly symmetrical to the charge distribution.
- Find the amount of charge enclosed.
- Plug it all into the equation above. Solve for E. Find the direction of E by thinking a bit about where it came from.

And, if you're still confused, check out the previous section. Gauss' Law is one of those things that becomes more clear by examples, for most ppl.

4.3 Continuous Charge Distributions

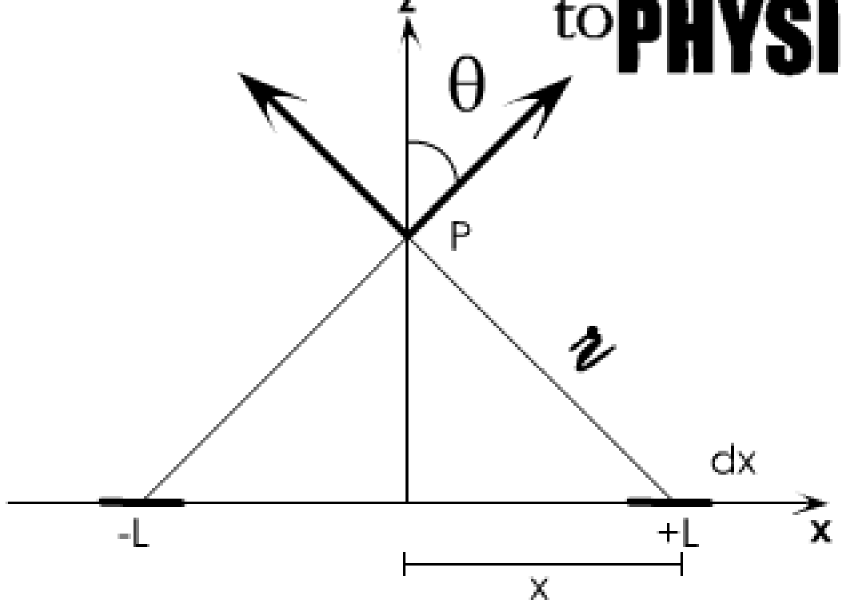
These are actually pretty easy if you remember your calculus. Anyway, the trick is this:

- Always remember your dear Coulomb's Law. To wit: $E = \frac{q}{4\pi\epsilon_0 r^2}$.
- Know that $dq = \lambda dl = \sigma da = \rho dV$. where $\lambda = \frac{charge}{length}$ and $\sigma = \frac{charge}{area}$ and $\rho = \frac{charge}{volume}$ where all denominators are "unit versions."
- Know your spherical/cylindrical coordinate dV and da's.

All of the results to be shown below can be verified with Gauss' Law. (They usually do it the other way, i.e., tell you to verify your Gauss' Law results with the equivalent but way more formidable and quite nasty continuous charge distribution calculations)

4.3.1 The Skinny Line Charges

Find the electric field a distance z above the midpoint of a straight line segment of length 2L, which carries a uniform line charge λ.



This is merely one of many equivalent diagrams with suitable parameters for integration of a Line Charge.

If you break this up into x and z components, you'll find that the x component cancels. (They're going in opposite directions.) Therefore, only the z component is nonzero. The electric field is thus only in the z direction.

$$d\vec{\mathbf{E}} = 2\frac{1}{4\pi\epsilon_0}\frac{\lambda dx}{{r_s}^2}\cos\theta\hat{\mathbf{z}} \tag{4.8}$$

(The denominator r_s^2 should be a script r, indicating the shortest Euclidean distance from the charge segment on the x axis to the arbitrary point P on the z axis, but i can't seem to TeX that.)

Note that $q = \lambda dx$, therefore this takes the form of Coulomb's Law. There is a 2 in front of this to indicate that the z component is additive. That is, you add the z component of the arrows on the left and right.

$\cos\theta$ is actually the projection of the field onto the z axis. Because of the way we have the integral set up, this projection is necessary to indicate the nature of the electric field, which does not cancel only in the z component.

$$\cos\theta = \frac{z}{r_s}, where r_s = \sqrt{z^2 + x^2} \qquad \text{from the Pythagorean Thrm.} \tag{4.9}$$

Thus, $\cos\theta = \frac{z}{\sqrt{z^2+x^2}}$:

$$\begin{aligned}
1.\ E &= \frac{1}{4\pi\epsilon_0}\int_0^L \frac{2\lambda z dx}{(z^2+x^2)^{3/2}} && (4.10)\\
2. \quad &= \frac{2\lambda z}{4\pi\epsilon_0}\frac{x}{z^2\sqrt{z^2+z^2}} && (4.11)\\
3. \quad &= \frac{1}{4\pi\epsilon_0}\frac{2\lambda L}{z\sqrt{z^2+L^2}} && (4.12)
\end{aligned}$$

1. Plugging everything back into the first equation. (4.13)
2. Factoring out the constants and integrating. (4.14)
3. Plugging in the limits and simplifying. (4.15)

This aims in the z direction. Thus, in vector form, it would be just: $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2\lambda L}{z\sqrt{z^2+L^2}}\hat{\mathbf{z}}$

That's basically all there is to continuous distributions. To do others, you just plug in different things for q and r_s. Find a suitable set of integration parameters, and it's all good!

4.4 Potential

sdfdfssd sdfsdf

Chapter 5

The Magnetism Introduction

Magnetism is about moving charges. The basic problem in Magnetism is to solve problems involving moving charges.

Electricity was just about plain ol'charges, not necessarily in motion. Any charge produces an electric field that effects the other charges in the set. But, the trick is magnetic fields exert forces on moving charges, and nothing else. (And it looks like moving charges produce magnetic fields.)

But... Waiiiiitt a sec! What about magnets, like that prize-winning collection sticking onto my refridgerator (outside)? They don't seem to be moving... Furthermore, they don't even seem to be charges – like whenever I play with them too much, my hair doesn't turn into a decent Afro. They probably contain some sort of magnetic field, as their stickin'-force seems to decrease the further away I put two of 'em together...

Good question. Your prize-winninge fridge magnet set actually contains magnetic fields on the microscopic atomic scale. There are tiny electrons flowing in a certain direction, each producing a magnetic field purely due to their motion. The field each produces all-together does not cancel out on the macroscopic scale, thus manifesting a magnetic field. The teeny tiny fields are thus all aligned in the same direction, thus magnetizing the matter.

For that matter, your prize-winning residue on the *inside* of your fridge probably also contains fields on the atomic level, but they probably all cancel out, by moving in some wantonly random directions, so as to not stick to each other in any other way except due to sheer non-field based stickiness. (Like most other sticky things. Take that annoying piece of gum sticking to your shoe.)

So, what I should have said was this: All moving charges produce magnetic fields, but because (most) things you can see without the aid of some hardcore microscope are all composed of multiple moving charges, they might not be magnetic overall because their charges might be moving in totally random directions, thus canceling out the fields on the wee bit teeny atomic level, way before you can see its overall macroscopic effects.

That's what Magnetism is about. Just nomad charges who don't like staying in one place for too long. We'll stalk them...

We'll start with something they call Magnetostatics next. (Statics? But.. I thought charges had to move to create magnetic fields...) You'll see, soon enough.

If you did badly in the first part of your E and M course, have no fear! Magnetism will be your redemption, basically because it'll help beef you up for the final - many of the calculations and stuff in Magnetism parallel the stuff done in Electricity. **This is your chance for fighting an enemy (magnetism) whose every move you've seen before (in electricity)! Don't let the curve kill you, again. Instead, try killing the curve for a decent revenge. XD**

If you did well in the first part of your E and M course, this second part will be like that special grape-flavored dashed with orange icing on the cake. **It'll be sw33t!**

Chapter 6

Magnetostatics

If you read the Intro to Magnetism, Magnetostatics might seem like a totally bogus word. The only things that produce magnetic fields are moving charges, so how can there be such a state as magneto-statics, which translates literally into "non-moving magnetism." ... Which seems to contradict itself.

Magnetostatics really refers to the state of currents that do not vary over time. (So, the statics is with respect to time.) These currents have electric fields that cancel out on a macroscopic level, because there is an approximately equal amount of negative and positive charges. Furthermore, these currents are all bound to the specific current line, and there are no free charges.

6.1 The Right Hand Rule

6.1.1 For Currents

Suppose there's an infinitely long current right in front of you, coming out of the ground, and going off into infinite space. The current is moving up towards the depths of the heavens. If you were to try to clasp onto it with your right hand. (Suppose you're omnipotent, and you don't get electrocuted, and you're still alive.) Now, your thumb is aligned (upwards) in the same direction as the current. Your fingers are curled around it. That's the only way you can clasp onto an infinite line current.

Your fingers are curled in the direction of the magnetic field, while your thumb points in the direction of the current. This is the Right Hand Rule for currents, and it's quite handy for determining the field direction for as long as you keep your right hand.

Don't you just love these "idealized situations?"

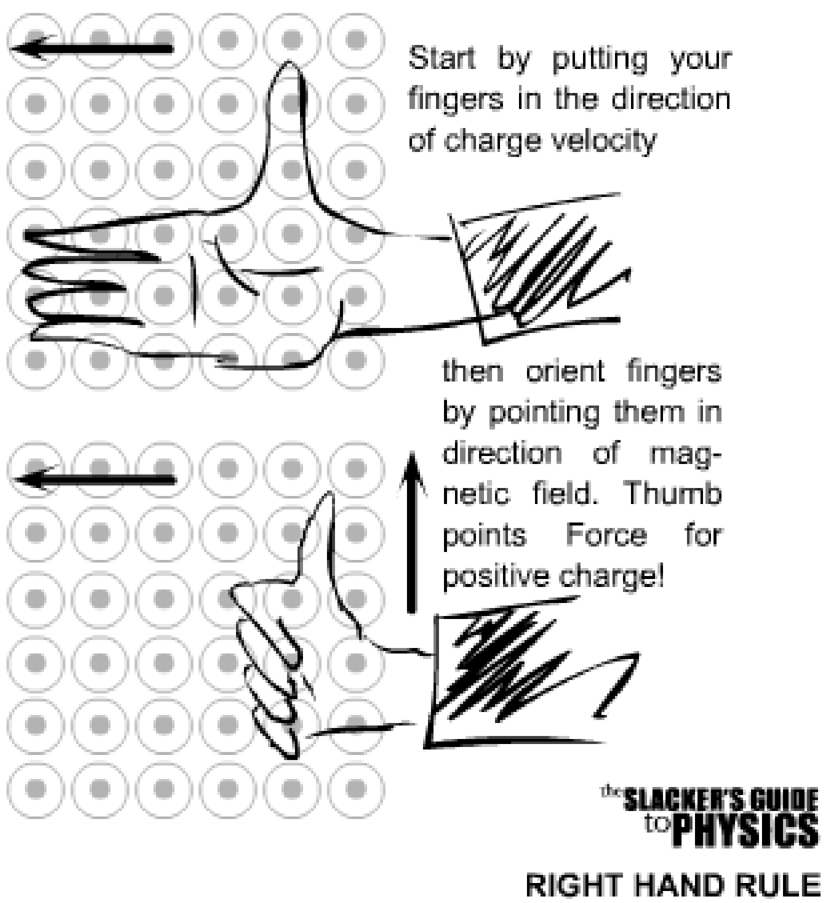
Now, this just might be a sign of obscentity in some eastern European hamlet, so don't go around waving at people with the right hand rule for currents when you visit Europe... (cf. One side of the "peace sign" is "the finger" in the UK)

6.1.2 For the Hermit Charge

Suppose the loner electron (or lone electron, if you're the chemmy type) is zooming rightwards through a region with a uniform magnetic field pointing in the direction coming out of the page, towards you.

Now electrons are the negative type of loners, so their charge value is negative. The general force for such loner electrons goes like:

$$\vec{F} = -q\vec{v} \times \vec{B} \tag{6.1}$$



Look at that short and lovely life line on the palm of the first figure in the diagram. The heart line is relatively stable, while the head lines points towards the life line. This is definitely the palm of a physicist. Your right hand stars in both pictures.

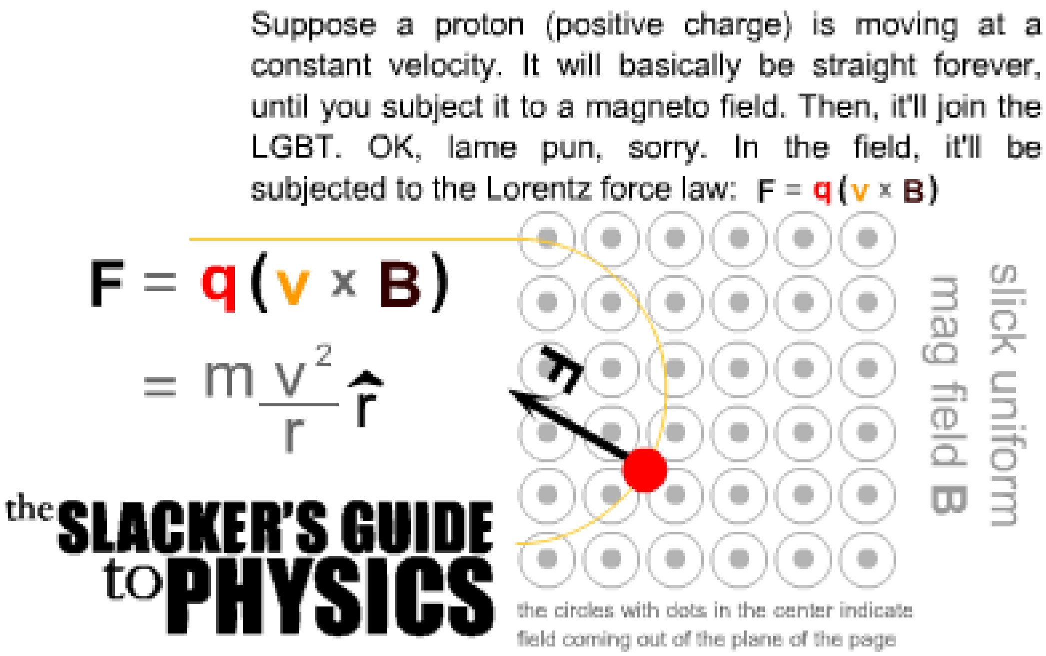
Let your fingers point in the direction of the velocity of the charge, then "move" your fingers by "orienting" them in the direction of the magnetic field. If you have a negative charge, the direction will be in the OPPOSITE direction of your thumb.

Contrast the difference, when you have a positive charge. (The force just goes in the same direction as your thumb)

$$\vec{F} = q\vec{v} \times \vec{B} \tag{6.2}$$

Check out the Right Hand Rule Interactive Animation on the website. http://emslacker.yosunism.com

Meanwhile, here's a static diagram in general for a positive charge. Note that the force points towards the center, as in centripetal motion:



6.2 Ampere's Law: The Gauss' Law of Magnetism

$$\vec{B} \cdot \vec{dl} = \mu_0 I_{in} \tag{6.3}$$

That's Ampere's Law. It's good for currents that are constant with respect to time. And, if you get vectors, there's really nothing to it. $\vec{B}$ refers to the magnetic field. $\vec{dl}$ refers to the Amperian path around the current. Note that dl is used differently than used in the Biot-Savert Law.

The Amperian path, like its Gaussian surface analog for Electricity, is always symmetrical to the thing producing it.

For example:

6.2.1 That Proverbial Infinite Line of Current

Recall the right hand rule for infinite current lines. You grab onto it by pointing your thumb in the direction of current flow, and hooking your (non-thumb) fingers over in their natural position after the thumb's been set. Your fingers curl in the direction of the magnetic field.

This direction is also the direction of the Amperian path. The direction of the Amperian path will determine the direction of the magnetic field. This goes by the same reasoning I put up for Gauss' Law. Because the left side of the equation involves a dot product, you know that all other components except the one that is in the direction of both dot product vectors should be zero. (Don't confuse yourself by looking into this problem in a way that's too philosophically profound here. It's really just the equation that you should be worried about.)

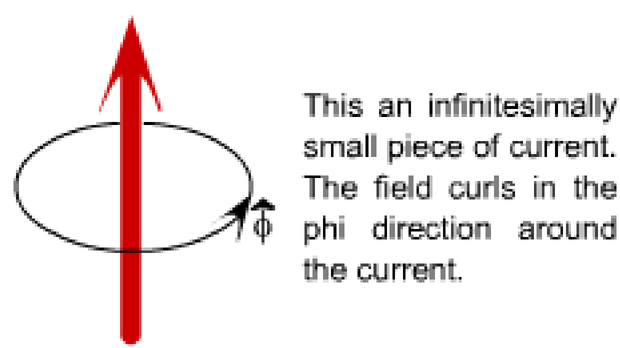
The right hand side involves a scalar current value multiplied by the constant of permeability for free space.

So, here's the problem, as stated formally:

Find the magnetic field of an infinite line current I.

OK, so from the stuff above, we know the direction of the magnetic field. (To wit: use the Right Hand Rule for Currents)

Because the current is cylindrical in the same way that an infinitely long thin soup can would be, the natural Amperian path that goes in approximately the same direction as your fingers would be a circle. The length of this Amperian path is $2\pi r$, which is the same as the circumference of a circle. The field is pointing in the ϕ direction, where ϕ goes from 0 to 2π.



Recall Ampere's Law: $\vec{B} \cdot \vec{dl} = \mu_0 I_{in}$

So, on the left hand side, you get: $\vec{B} \cdot 2\pi r\hat{\phi}$ Dot them together, and you get $B(2\pi r)$

On the right hand side, you just get the default: $\mu_0 I$

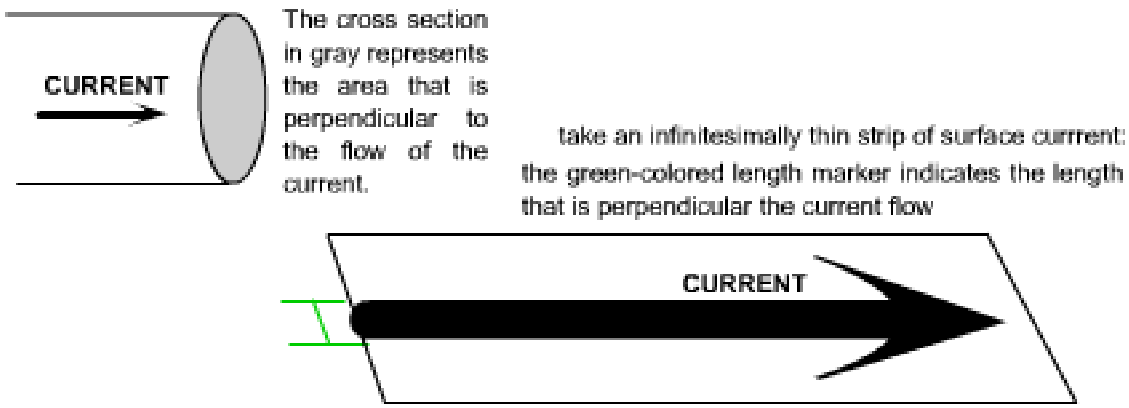
You then solve for B. You get: $\vec{B} = \frac{\mu_0 I\hat{\phi}}{2\pi r}$

6.2.2 Current Distributions

There's the volume current distribution: $J \equiv \frac{I}{A\perp}$

And then there's also the surface current distribution: $K \equiv \frac{I}{l\perp}$

Notice that I've placed a $\perp$ subscript by both the Area and the length. This means that I want the area component that is perpendicular to the flow. Check out the pic:



6.2.3 The Fat Arse Infinite Line Current

Just like how evil professors can make you do evil integrals to find the charge enclosed for Gauss' Law, they can do the same for currents in Ampere's Law. Like in Gauss' Law, the left hand side is dependent only on the geometry of the system. Thus, no matter what wacky current distribution you have, as long as it's along a right cylindrical wire, the left hand side will be the same as that shown above.

Given a volume current distribution, the right hand side can be determined by:

$$I = \int J da_{\perp} \tag{6.4}$$

where da is the area perpendicular to the current flow.

Similarly, given a surface current distribution, the right hand side can be determined by:

$$I = \int K dl_{\perp} \tag{6.5}$$

where dl is the length perpendicular to the current flow.

Anyway, here's a sample problem: **Find the $\vec{B}$ everywhere produced by a volume current density $J = kr$ distributed in an infinitely long "thick arse wire" of radius R.**

For $r < R$, that is, within the "thick arse wire", we can start with the right hand side: $I = \int_0^r \int_0^{2\pi} kr(rd\phi dr) = k\frac{r^3}{3}2\pi$

Note the usage of polar coordinates, which is appropriate for integrating cross sections involving circles.

You use what we found above for the left hand side for Ampere's law: $B(2\pi r)$

Then, you set it equal to each other according to Ampere's law: $B(2\pi r) = \mu_0 k\frac{r^3}{3}2\pi$

Then, you solve for B... You get: $B = \mu_0 k\frac{r^2}{3}\phi$ The current is going in the z direction in cylindrical coordinates, or in other words, away from you. Thus, your magnetic field would be in the ϕ direction.

For $r > R$, that is outside the "thick arse wire", we can start with the right hand side, again: $I = \int_0^R \int_0^{2\pi} kr(rd\phi dr) = k\frac{R^3}{3}2\pi$

Note that the top limit ends at the radius of the circle, basically because the problem defines current to be flowing only within this infinitely long right cylindrical thing.

You use what we found above for the left hand side for Ampere's law: $B(2\pi r)$

Then, you set it equal to each other according to Ampere's law: $B(2\pi r) = \mu_0 k\frac{R^3}{3}2\pi$

Then, you solve for B... You get: $B = \mu_0 k\frac{R^3}{3r}\phi$ The current is going in the z direction in cylindrical coordinates, or in other words, away from you. Thus, your magnetic field would be in the ϕ direction.

Here's another problem: Same thing as above, except with uniform volume current. Total current is I. Radius is R.

The field on the outside is trivial; it's the same as that for a thin wire.

The field on the inside can be done in pretty much the same way we found that for uniform volume charges back in Electricity:

We use a bit of dimensional analysis, and we get: $\frac{I_{in}}{\pi r^2} = \frac{I}{\pi R^2} = J$ Solve for I_{in}, and ta-da, plug into Ampere's law, and you get this: $B = \mu_0 I\frac{R^2 r}{2\pi}$;

Appendix A

Appendix 1

A.1 Components: Breaking Up the Multidimensional

Components may seem imposing, but that's only *if you let them be*. They let you shatter seemingly complex two or three dimensional problems into some one dimensional common sense. In a way, components are actually like the x and y coordinates, you have when, for example, you plot $(4, 6)$. It's actually something you've seen a gazillion times before disguised in a seemingly scary form.

If you don't understand components, that's probably because you don't quite get the trig. Thus, this section will start with "the trig."

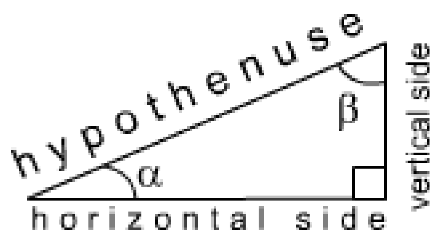
If you can solve the following problem fairly easily and are reasonably good with basic inclined plane problems, then you should skip this whole component, er, section.

Suppose plane A were inclined at an angle of α from the absolutely flat ground. Then, suppose that plane B were inclined at an angle of α on or with respect to plane A. In other words, the two planes form alternate interior angles, and they're flat with respect to the same ground. Plane B is "growing" out of plane A; the side of the triangle denoting plane B has one angle α and another right angle (90 degrees). Given the fact that the hypotenuse of the triangle formed by plane B is h, find the horizontal and vertical components with respect to the variables h, and α. The format of this question should remind you of an old friend from mechanics...

The answer to this problem, along with a nifty diagram is available later in this section. In order to skip it without fear that you'll miss too much, you need to solve this problem correctly... NOW. Otherwise, my dear reader, please read on and ignore this problem until the right time comes.

A.1.1 Conquering the Trig

To begin mathematics, mathematicians had to define things. That's more or less why the three basic trig ratios (sin, cos, and tan) are known as what they are.



$$\sin\alpha \equiv \frac{\mathbf{o}\mathit{pposite}}{\mathbf{h}\mathit{ypothenuse}} \tag{A.1}$$

$$\cos\alpha \equiv \frac{\mathbf{a}\mathit{djacent}}{\mathbf{h}\mathit{ypothenuse}} \tag{A.2}$$

$$\tan\alpha \equiv \frac{\mathbf{o}\mathit{pposite}}{\mathbf{a}\mathit{djacent}} \tag{A.3}$$

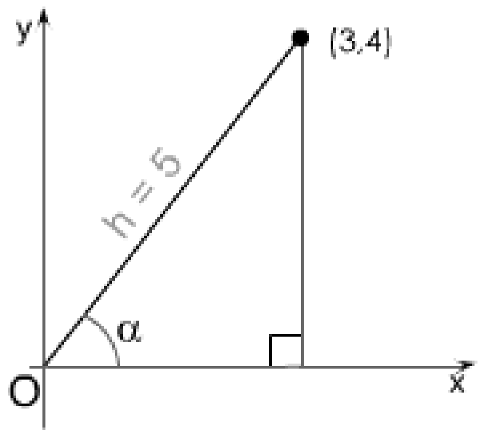
The opposite side, in this case, would be the vertical side. The adjacent side, in this case, would be the horizontal side. The opposite/adjacent demarcations are with respect to the angle alpha (α).

But, you have to note something weird here. If I were to have β as the angle, the following would be true: The opposite side would be the *horizontal side*, while the adjacent the *vertical side*. Why? The side that is adjacent (touching) to the angle beta (β) is the vertical side, and so on. *Thus, it is important that you remember the general rules defined above, rather than assuming that cosines always involve the horizontal divided by the hypothenuse.*

You can remember this by the nifty mneumonics to follow:

- When asked whether he would repent his sins, Galileo tried to avoid answering the question via the following historic quote: "OH, SIN.... AH–'COS .... TAN Ohh.. Ahh.." (where the O in OH stands for Opposite, while the H stands for Hypotenuse, etc.)
- With every HYPe, there's an OP for SIN, COS of an AD potraying OPportunities and ADventures with some crazy TAN people.
- There's always the old SOA-CAH-TOA, if you like that one better.

A.1.2 Facing the Components



In a way, components are actually like the x and y coordinates, you have when, for example, you plot $(3, 4)$. The x component is 3, while the y component is 4. You would get a triangle if you were to draw a straight line from the origin to the point, letting the x and y axes become the other two sides. You would use the Pythagorean Theorem $h = \sqrt{a^2 + b^2}$ to find the length of the slanted part of the triangle.

In this case, the length of the slanted part - or the hypotenuse - would be $h = \sqrt{3^2 + 4^2} = 5$

Now that you know the hypothenuse, you can easily find the components in terms of the angle α.

Recall the definition for sin and cos from the previous page.

$\sin\alpha \equiv \frac{\mathbf{o}pposite}{\mathbf{h}ypothenuse}$ and $\cos\alpha \equiv \frac{\mathbf{a}djacent}{\mathbf{h}ypothenuse}$

Relative to the diagram above, sin and cos can be represented as:

$\sin\alpha = \frac{y}{h}$ and $\cos\alpha = \frac{x}{h}$

To solve for y or x, you would just multiply both sides by h:

$y = h\sin\alpha$ and $x = h\cos\alpha$

You can solve for alpha by pluging in the x and y and h values for this particular point:

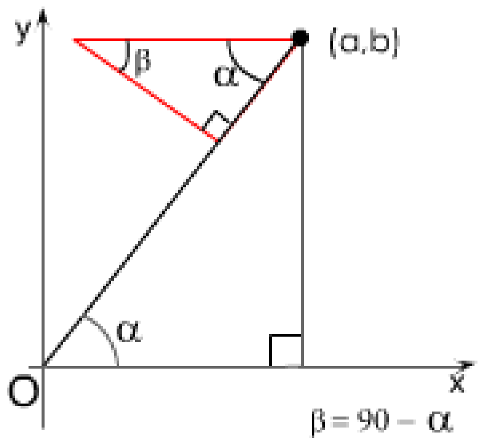
$4 = 5\sin\alpha \Rightarrow \alpha = \arcsin 4/5$ and$3 = 5\cos\alpha \Rightarrow \alpha = \arccos 3/5$

Thus, alpha is approximately 53.13 degrees, if you type in the expression above into your calculator.

This is basically all there is breaking up a multidimensional thing into components. You have to do it with respect to the right angle and hypothenuse. This can be done in three dimensions, too, but there will likely be more than one angle you'll have to plug into the sin's and cos's.

A.1.3 Components on Inclined Planes

Here's that problem I promised to solve earlier:



To requote:

Suppose plane A were inclined at an angle of α from the absolutely flat ground. Then, suppose that plane B were inclined at an angle of α on or with respect to plane A. In other words, the two planes form alternate interior angles, and they're flat with respect to the same ground. Plane B is “growing” out of plane A; the side of the triangle denoting plane B has one angle α and another right angle (90 degrees). Given the fact that the hypotenuse of the triangle formed by plane B is h, find its horizontal and vertical components with respect to the variables h, and α. The format of this question should remind you of an old friend from mechanics...

We know that that that particular angle on the other triangle is also α because alternate internal angles are equal (geometry)... And, we know they're both right triangles. Thus, $\beta = 90 - \alpha$

Now, we apply our trig rules. the "horizontal" (parallel to hypothenuse of black triangle) part of triangle B (the red one) would be $h \sin \beta$, while the vertical part (normal to hypothenuse of black triangle) would be $h \cos \beta$.

You've just flipped over your coordinate system to be inclined-plane friendly. Your normal force and your frictional force are now naturally represented! Well, we're assuming the force is applied along the hypothenuse of the red triangle...